Lesson 24

Recall that we can find relative extrema of a function of two variables, $z = f(x, y)$ using the second derivative test:

1. Find the points (a, b) where $f_x(a, b)$ ***and*** $f_y(a, b) = 0$. These are our critical points.
2. Let $D = f_{xx}(a, b)f_{yy}(a, b) - [f_{xy}(a, b)]^2$.
   If $D > 0$ and $f_{xx}(a, b) > 0$ then there is a local minimum at (a, b).
   If $D > 0$ and $f_{xx}(a, b) < 0$ then there is a local maximum at (a, b).
   If $D < 0$ then there is a saddle point at (a, b).
   If $D = 0$ then the test fails.

1. Find all relative extrema of $f(x, y) = 9x^2 + 9y^2 + 12xy$.

(a, b)	$D(a, b)$	$f_x(a, b)$	Type of Extrema

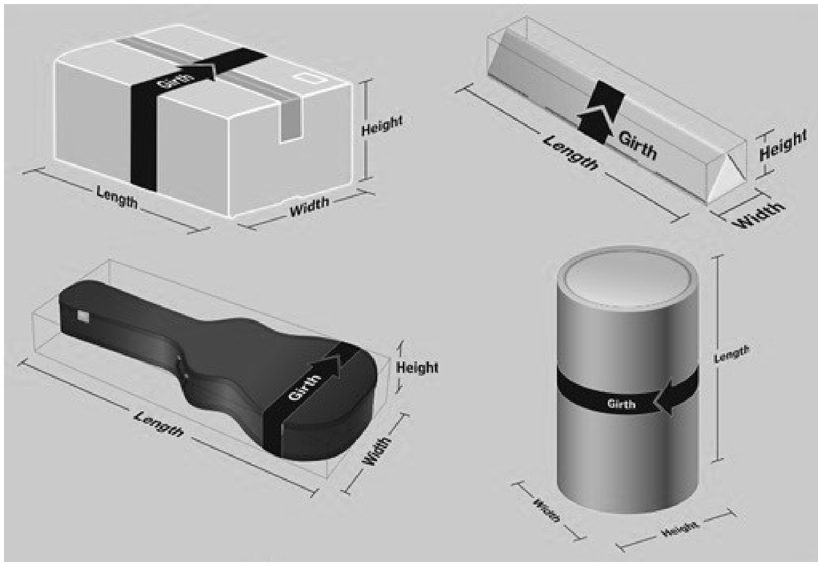
2. (Calc 1 Review) A 12 ounce pop can has a volume of about 22 in^3. Coca-Cola has hired you to find the dimensions that minimize the amount of material used.

   (a) Set up an equation for the amount of material used in terms of r and h.

   (b) Rewrite the volume in terms of one variable.

   (c) Find the dimensions that minimize the amount of material used.

3. The post office will accept packages whose combined length and girth is at most 108 inches. (Length is the longest dimension and girth is the maximum distance around the package perpendicular to the length.) What is the largest volume that can be sent in a rectangular package?



(a) Set up an equation for the volume in terms of ℓ, w, and h.

(b) Rewrite the volume in terms of two variables.

(c) Find the maximum volume that can be sent.

4. Maximize the volume of a rectangular box if you have \$120 to spend, the material for the top costs \$8 per square foot, the material for the sides costs \$3 per square foot, and the material for the bottom costs \$6 per square foot.

   (a) Set up an equation for the volume in terms of ℓ, w, and h.

   (b) Rewrite the volume in terms of two variables.

   (c) Find the maximum volume.